Chapter 0 Preliminaries

Before we start, the following are topics from high school that you'll need to use frequently:

- **Factorization** and **solving polynomial equations**
- **Laws of exponents** and **laws of logarithms**
- **Mensuration**, **areas** and **volumes** of simple geometric objects
- The **coordinate plane**, slopes and equations of **lines**, equations of **circles**
- **Trigonometry**

Make sure you don't forget what you have learnt about these tools. Apart from the above topics, we will introduce in this chapter a few other preliminary topics which are particularly important.

1. Basic set theory

In the subject of calculus, the fundamental objects that we study are **functions**. We need some basic language in **set theory** before we can define what a function is.

A ***bold and italicized phrase*** in a definition indicates the phrase that is being defined.

Definition 0.1 A ***set*** is a collection of objects. The objects collected in a set are called ***elements*** of the set.

- Each element of a set A is said to ***belong to*** A. In symbols we write $x \in A$ if an object x belongs to the set A, and we write $x \notin A$ if x does not belong to A.
- Two sets A and B are said to be ***equal***, written $A = B$, if they contain the same elements.

Example 0.2 There are various ways to denote the same set, for example:

- List all its elements (in any order),
$$A = \{1, 3, 5, 7, 9\}.$$
- Write down conditions that its elements satisfy,
$$A = \{x: 0 < x < 10 \text{ and } x \text{ is an odd number}\}.$$

In this example, we have $1 \in A$, $3 \in A$, $7 \in A$, but $2 \notin A$, $6 \notin A$, $\sqrt{2} \notin A$ and $\pi \notin A$.

Example 0.3 Rewrite each of the following sets by listing all its elements.

(a) $A = \{x: 0 < x < 5 \text{ and } x \text{ is an integer}\}$

(b) $B = \{x: (x-2)(x-3) = 0\}$

(c) $C = \{x: x \text{ is a vowel in the English alphabet}\}$

Instead of the form {object: conditions}, some authors denote a set in the form {object | conditions}, which is also fine.

Solution:

(a) $A = \{1, 2, 3, 4\}$

(b) $B = \{2, 3\}$

(c) $C = \{\text{a, e, i, o, u}\}$

Example 0.4 The order, arrangement and repetition of the elements in a set is irrelevant, e.g.

- The sets $\{1,1,1,2,2,3\}$ and $\{1,2,3\}$ are equal;
- The sets $\{1,2,3,4,5\}$ and $\{2,1,4,5,3\}$ are equal.

Definition 0.5 The ***empty set*** contains nothing and is denoted by $\emptyset$,

$$\emptyset := \{\ \}.$$

The ***set of all real numbers*** is denoted by $\mathbb{R}$,

$$\mathbb{R} := \{x: x \text{ is a real number}\}.$$

When we introduce a **new notation**, we write the notation on the left-hand side of ":=", and its meaning on the right-hand side of ":=".

In calculus, we mainly deal with sets that consist of (some) real numbers only. The following is an important class of "connected" sets, called **intervals**.

Definition 0.6 Let $a \leq b$ be two real numbers. Then the following sets are called ***intervals***:

Open intervals	***Closed intervals***	***Half-open intervals***
$(a,b) := \{x \in \mathbb{R}: a < x < b\}$	$[a,b] := \{x \in \mathbb{R}: a \leq x \leq b\}$	$(a,b] := \{x \in \mathbb{R}: a < x \leq b\}$
$(a,+\infty) := \{x \in \mathbb{R}: x > a\}$	$[a,+\infty) := \{x \in \mathbb{R}: x \geq a\}$	$[a,b) := \{x \in \mathbb{R}: a \leq x < b\}$
$(-\infty,b) := \{x \in \mathbb{R}: x < b\}$	$(-\infty,b] := \{x \in \mathbb{R}: x \leq b\}$	

Here "$+\infty$" and "$-\infty$" are just **symbols**. They are NOT real numbers!

- The real numbers a and b are called the ***left end-point*** and the ***right end-point*** of the corresponding intervals.
- Every other number contained in each interval is said to be in the ***interior*** of that interval.
- The ***width*** of an interval with left end-point a and right end-point b is the number $b-a$.

We can do **operations** on two sets to produce a new set. These operations include the union, the intersection and the complement.

Definition 0.7 Let A and B be sets.

- The ***intersection*** of A and B is the set
$$A \cap B := \{x: x \in A \text{ and } x \in B\}.$$
- The ***union*** of A and B is the set
$$A \cup B := \{x: x \in A \text{ or } x \in B\}.$$
- The ***complement*** of B in A is the set
$$A \setminus B := \{x: x \in A \text{ and } x \notin B\}.$$

We say that A and B are ***disjoint*** if their intersection is empty, i.e. $A \cap B = \emptyset$.

Example 0.8 Each of the following sets can be expressed as a union of disjoint intervals:

(a) $(-3,5) \cap [4,7] = [4,5)$, $\quad (-3,5) \cup [4,7] = (-3,7]$

(b) $[2,5] \cap \big((1,3) \cup (4,6)\big) = [2,3) \cup (4,5]$

(c) $\mathbb{R} \setminus \big([0,2] \cap (1,3)\big) = \mathbb{R} \setminus (1,2] = (-\infty,1] \cup (2,+\infty)$

2. Inequalities in one unknown

In the previous section when we defined intervals, we made use of inequalities in one unknown. Let us review some facts and tools for solving **inequalities in one unknown**.

Theorem 0.9 *Let a, b, c be real numbers.*
(i) *If $a < b$, then $a + c < b + c$ and $a - c < b - c$.*
(ii) *If $a < b$ and $c > 0$, then $ac < bc$.*
(iii) *If $a < b$ and $c < 0$, then $ac > bc$.*

Remark 0.10 When multiplying or dividing both sides of an inequality by something, watch out whether that something is positive or negative!

Example 0.11 Solve the inequality
$$4x - 3 < 2x + 5.$$

Solution:
Adding $(3 - 2x)$ on both sides of the given inequality, we obtain
$$(4x - 3) + (3 - 2x) < (2x + 5) + (3 - 2x),$$
i.e. $2x < 8$. Dividing both sides by 2 we obtain $x < 4$. Therefore the solutions to the given inequality are all those $x \in (-\infty, 4)$.

Example 0.12 Solve the inequality
$$\frac{4}{x - 3} \leq 2.$$

Solution:
Note that $x - 3 \neq 0$ because $x - 3$ appears in the denominator of an expression.

⊙ If $\underline{x - 3 > 0}$, then multiplying $x - 3$ on both sides the inequality becomes
$$4 \leq 2(x - 3).$$
Rearranging this inequality we get $2x \geq 10$, and so $x \geq 5$. Combining with the assumption that $x - 3 > 0$, we obtain ($x \geq 5$ and $x > 3$), which is the same as just $x \geq 5$.

⊙ If $\underline{x - 3 < 0}$, then multiplying $x - 3$ on both sides the inequality becomes
$$4 \geq 2(x - 3).$$
Rearranging this inequality we get $2x \leq 10$, and so $x \leq 5$. Combining with the assumption that $x - 3 < 0$, we obtain ($x \leq 5$ and $x < 3$), which is the same as just $x < 3$.

Now combining both cases, the solutions to the inequality are all those $x \in (-\infty, 3) \cup [5, +\infty)$.

Alternative solution:

The given inequality can be rewritten as $2 - \frac{4}{x-3} \geq 0$, or after clearing the denominators,

$$\frac{2x-10}{x-3} \geq 0.$$

Note that as x varies, the numerator and the denominator have "sign changes" when $x = 5$ and when $x = 3$ respectively. Let's analyze the sign of the whole expression on the left-hand side:

x	$(-\infty, 3)$	$x = 3$	$(3, 5)$	$x = 5$	$(5, +\infty)$
$2x - 10$	$-$	$-$	$-$	0	$+$
$x - 3$	$-$	0	$+$	$+$	$+$
$\frac{2x-10}{x-3}$	$+$	undefined	$-$	0	$+$

From this table, we see that the inequality $\frac{2x-10}{x-3} \geq 0$ holds if and only if $x \in (-\infty, 3) \cup [5, +\infty)$.

So the solutions to the given inequality are all those $x \in (-\infty, 3) \cup [5, +\infty)$. Set-theoretically we may write

$$\left\{x \in \mathbb{R}\colon \frac{4}{x-3} \leq 2\right\} = (-\infty, 3) \cup [5, +\infty).$$

Example 0.13 Solve the inequality

$$\frac{(x+1)^2 + 3}{(x+2)(x-4)} \leq 0.$$

Solution:
Note that in the expression on the left-hand side, the **numerator** $(x+1)^2 + 3$ is always ≥ 3, so the numerator is **always positive** in particular. Now let's analyze the sign of the whole expression on the left-hand side:

x	$(-\infty, -2)$	$x = -2$	$(-2, 4)$	$x = 4$	$(4, +\infty)$
$x + 2$	$-$	0	$+$	$+$	$+$
$x - 4$	$-$	$-$	$-$	0	$+$
$\frac{(x+1)^2+3}{(x+2)(x-4)}$	$+$	undefined	$-$	undefined	$+$

From this table, we deduce that the solutions to the given inequality

$$\frac{(x+1)^2 + 3}{(x+2)(x-4)} \leq 0$$

are all those $x \in (-2, 4)$. Set-theoretically we may write

$$\left\{x \in \mathbb{R}\colon \frac{(x+1)^2 + 3}{(x+2)(x-4)} \leq 0\right\} = (-2, 4).$$

3. Absolute values

Definition 0.14 Let a be a real number. The ***absolute value*** of a, denoted as $|a|$, is defined by

$$|a| = \begin{cases} -a & \text{if } a < 0 \\ a & \text{if } a \geq 0 \end{cases}.$$

Example 0.15 $|5| = 5$ and $|-5| = 5$.

Corollary 0.16 *Let a be a real number. Then*

$$\sqrt{a^2} = |a|.$$

In particular, we must have $|a| \geq 0$.

Notice how $\sqrt{x^2}$ and $(\sqrt{x})^2$ are different from each other.

Remark 0.17 Let a and b be real numbers. The quantity $|a-b|$ represents the "distance" between the numbers a and b. We have

$$|a-b| = \begin{cases} b-a & \text{if } a < b \\ a-b & \text{if } a \geq b \end{cases}.$$

Theorem 0.18 *Let a and b be real numbers. Then*

(i) $|ab| = |a||b|$

(ii) $|a^2| = |a|^2 = a^2$

(iii) $|-a| = |a|$

(iv) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ *if* $b \neq 0$.

Remark 0.19 In order to remove the absolute value sign from an expression $|*|$, we first check whether the expression $*$ inside the absolute value sign is negative or not.

⦿ If $* < 0$, then we write an extra negative sign after removing the absolute value sign, i.e.

$$|*| = -*;$$

⦿ if $* \geq 0$, then we do nothing after removing the absolute value sign, i.e.

$$|*| = *.$$

Example 0.20 For each of the following inequalities, find all the real numbers x which satisfy the inequality:

(a) $|2x-5| > 7$

(b) $|x-2| + |2x+1| \leq 4$

Solution:

(a) ⊙ If $2x-5\ge 0$, i.e. if $x\ge\frac{5}{2}$, then the inequality becomes $2x-5>7$, and so $x>6$.

Combining $x\ge 5/2$ and $x>6$, we see that every $x>6$ is a solution to the inequality.

⊙ If $2x-5<0$, i.e. if $x<\frac{5}{2}$, then the inequality becomes $-(2x-5)>7$, so $x<-1$.

Combining $x<5/2$ and $x<-1$, we see that every $x<-1$ is also a solution.

Therefore the solutions to the inequality consists of all those $x\in(-\infty,-1)\cup(6,+\infty)$.

(b) ⊙ If $x\in\left(-\infty,-\frac{1}{2}\right)$, then both $x-2<0$ and $2x+1<0$, and the inequality becomes

$$-(x-2)-(2x+1)\le 4,$$

so $x\ge -1$. Combining with $x<-\frac{1}{2}$, we see that every $x\in\left[-1,-\frac{1}{2}\right)$ is a solution.

⊙ If $x\in\left[-\frac{1}{2},2\right)$, then $x-2<0$ but $2x+1\ge 0$, and the inequality becomes

$$-(x-2)+(2x+1)\le 4,$$

so $x\le 1$. Combining with $-\frac{1}{2}\le x<2$, we see that every $x\in\left[-\frac{1}{2},1\right]$ is a solution.

⊙ If $x\in[2,+\infty)$, then both $x-2\ge 0$ and $2x+1\ge 0$, and the inequality becomes

$$(x-2)+(2x+1)\le 4,$$

so $x\le 5/3$. But there is no x which satisfies both $x\ge 2$ and $x\le 5/3$, so this case does not give any solutions.

Therefore the solutions to the inequality consists of all those $x\in[-1,1]$.

When handling equations or inequalities that involve absolute values, the following fact is useful.

Corollary 0.21 *Let x be a real number and c be a <u>non-negative</u> real number.*

(i) $|x|<c$ *if and only if* $-c<x<c$.

(ii) $|x|=c$ *if and only if either* $x=c$ *or* $x=-c$.

(iii) $|x|>c$ *if and only if either* $x>c$ *or* $x<-c$.

If c were a negative real number in the above corollary, then it would be obvious that $|x|<c$ is always impossible, $|x|=c$ is always impossible and $|x|>c$ is always true.

Example 0.22 Solve the inequality

$$\left|3-\frac{4}{x}\right|<1.$$

Solution:

Note that $\left|3-\frac{4}{x}\right|<1$ if and only if $-1<3-\frac{4}{x}<1$, i.e.

$$4-\frac{4}{x}>0 \qquad \text{and} \qquad 2-\frac{4}{x}<0.$$

⊙ The first inequality $4-\frac{4}{x}>0$ is the same as $\frac{4(x-1)}{x}>0$. From the following sign table

x	$(-\infty,0)$	$x=0$	$(0,1)$	$x=1$	$(1,+\infty)$
$x-1$	$-$	$-$	$-$	0	$+$
x	$-$	0	$+$	$+$	$+$
$\frac{4(x-1)}{x}$	$+$	undefined	$-$	0	$+$

we deduce that $4-\frac{4}{x}>0$ if and only if $x\in(-\infty,0)\cup(1,+\infty)$.

⊙ The second inequality $2-\frac{4}{x}<0$ is the same as $\frac{2(x-2)}{x}<0$. From the following sign table

x	$(-\infty,0)$	$x=0$	$(0,2)$	$x=2$	$(2,+\infty)$
$x-2$	$-$	$-$	$-$	0	$+$
x	$-$	0	$+$	$+$	$+$
$\frac{2(x-2)}{x}$	$+$	undefined	$-$	0	$+$

we deduce that $2-\frac{4}{x}<0$ if and only if $x\in(0,2)$.

Finally, the solutions to the original inequality $\left|3-\frac{4}{x}\right|<1$ consist of all those x which belong to both $(-\infty,0)\cup(1,+\infty)$ and $(0,2)$. Therefore the solutions are all those $x\in(1,2)$.

Theorem 0.23 *Let a and b be real numbers. Then*
(i) $-|a|\le a\le|a|$,
(ii) (*Triangle inequality*) $|a+b|\le|a|+|b|$.

Proof. (i) is obviously true. Now given real numbers a and b, (i) implies that

$$-|a|\le a\le|a| \qquad \text{and} \qquad -|b|\le b\le|b|;$$

thus we have

$$-(|a|+|b|)\le a+b\le|a|+|b|$$

and so $|a+b|\le|a|+|b|$ by Corollary 0.21. Therefore (ii) follows. ■

4. Sigma notation

In calculus, we will often evaluate a **sum of many numbers** in which the terms are almost identical to each other, with the only difference in the value of a running index. To simplify the notations of such sums, we use the following **Sigma notation**.

Definition 0.24 (Sigma notation) Let $a_1, a_2, \dots, a_n$ be numbers. Then we denote

$$\sum_{k=1}^{n} a_k := a_1 + a_2 + \cdots + a_n.$$

Remark 0.25 The sum $\sum_{k=1}^{n} a_k$ depends only on n, but does not depend on the "summation index" k. In fact, we can replace k by any symbol without changing the meaning of the notation:

$$\sum_{k=1}^{n} a_k = \sum_{j=1}^{n} a_j = \sum_{m=1}^{n} a_m \,.$$

We say that the summation index k is a ***dummy variable*** in the sum $\sum_{k=1}^{n} a_k$, which means that the overall sum actually contains no such variable.

Lemma 0.26 *Let $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n, c$ be numbers. Then we have the following:*

(i) $\sum_{k=1}^{n} c = cn$

(ii) $\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$

(iii) $\sum_{k=1}^{n} c a_k = c \sum_{k=1}^{n} a_k$

Remark 0.27 It is important to note that the following equalities are in general **NOT** true.

$$\sum_{k=1}^{n} a_k b_k \neq \left(\sum_{k=1}^{n} a_k\right)\left(\sum_{k=1}^{n} b_k\right) \qquad \text{and} \qquad \sum_{k=1}^{n} \frac{a_k}{b_k} \neq \frac{\sum_{k=1}^{n} a_k}{\sum_{k=1}^{n} b_k}.$$

Remark 0.28 The summation index can also start from any integer other than 1. We may also do a "shifting" in the summation index without affecting the overall sum. As an example, every summation of the form $\sum_{k=1}^{n} a_k$ can be rewritten as

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} a_{k+1}$$

because both sides represent the same sum $a_1 + a_2 + \cdots + a_n$.

Theorem 0.29 *Let n be a positive integer. Then we have the following identities:*

(i) (Sum of the first n positive integers)

$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

(ii) (Sum of the first n squares)

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) (Sum of the first n cubes)

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

Proof. To prove (i), we note that

$$k(k+1) - (k-1)k = 2k$$

for each positive integer k, so we have

$$\sum_{k=1}^{n} k = \sum_{k=1}^{n} \frac{k(k+1)-(k-1)k}{2} = \sum_{k=1}^{n} \frac{k(k+1)}{2} - \sum_{k=1}^{n} \frac{(k-1)k}{2}$$

$$= \sum_{k=1}^{n} \frac{k(k+1)}{2} - \sum_{k=0}^{n-1} \frac{k(k+1)}{2} = \frac{n(n+1)}{2} - \frac{0(0+1)}{2} = \frac{n(n+1)}{2}.$$

To prove (ii), we note that

$$k(k+1)(k+2) - (k-1)k(k+1) = 3(k^2 + k)$$

for each positive integer k, so using (i) we get

$$\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n} \left[\frac{k(k+1)(k+2)-(k-1)k(k+1)}{3} - k\right]$$

$$= \sum_{k=1}^{n} \frac{k(k+1)(k+2)}{3} - \sum_{k=1}^{n} \frac{(k-1)k(k+1)}{3} - \sum_{k=1}^{n} k$$

$$= \sum_{k=1}^{n} \frac{k(k+1)(k+2)}{3} - \sum_{k=0}^{n-1} \frac{k(k+1)(k+2)}{3} - \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(n+2)}{3} - \frac{0(0+1)(0+2)}{3} - \frac{n(n+1)}{2} = \frac{n(n+1)(2n+1)}{6}.$$

The proof of (iii) is similar and is left as an exercise. ■

Theorem 0.30 (Geometric sum) *Let $r \neq 1$ be a number and n be a positive integer. Then*

$$\sum_{k=0}^{n-1} r^k = \frac{r^n - 1}{r - 1}.$$

Proof. Let $s := \sum_{k=0}^{n-1} r^k$. Then $rs = \sum_{k=0}^{n-1} r^{k+1} = \sum_{k=1}^{n} r^k$, so

$$rs - s = \sum_{k=1}^{n} r^k - \sum_{k=0}^{n-1} r^k = r^n - r^0 = r^n - 1.$$

Rearranging we get $s = \frac{r^n - 1}{r - 1}$. ■

Example 0.31

(a) Find the sum of squares of the first n positive even numbers.

(b) Evaluate the sum $50 \cdot 52 + 51 \cdot 53 + 52 \cdot 54 + \cdots + 99 \cdot 101$.

(c) Evaluate the sum $3 + 6 + 12 + 24 + \cdots + 3072$.

Solution:

(a) The sum of squares of the first n positive even numbers is given by

$$\begin{aligned}\sum_{k=1}^{n} (2k)^2 &= \sum_{k=1}^{n} 4k^2 = 4\sum_{k=1}^{n} k^2 \\ &= 4\left[\frac{n(n+1)(2n+1)}{6}\right] \\ &= \frac{2n(n+1)(2n+1)}{3}.\end{aligned}$$

(b)

$$\begin{aligned}&50 \cdot 52 + 51 \cdot 53 + 52 \cdot 54 + \cdots + 99 \cdot 101 \\ &= \sum_{k=51}^{100} (k-1)(k+1) = \sum_{k=51}^{100} (k^2 - 1) \\ &= \sum_{k=51}^{100} k^2 - \sum_{k=51}^{100} 1 = \left(\sum_{k=1}^{100} k^2 - \sum_{k=1}^{50} k^2\right) - \sum_{k=51}^{100} 1 \\ &= \frac{(100)(101)(201)}{6} - \frac{(50)(51)(101)}{6} - 50 \\ &= 295375\end{aligned}$$

(c)

$$\begin{aligned}
&3+6+12+24+\cdots+3072\\
&=3+3\cdot 2+3\cdot 2^2+3\cdot 2^3+\cdots+3\cdot 2^{10}\\
&=\sum_{k=0}^{10}(3\cdot 2^k)=3\sum_{k=0}^{10}2^k=3\cdot\frac{2^{11}-1}{2-1}\\
&=3\cdot 2047=6141.
\end{aligned}$$

As a by-product of the geometric sum we obtain the following identity, whose $n=2$ and $n=3$ cases should be well-known from high school.

Corollary 0.32 *Let a and b be numbers and n be a positive integer. Then*

$$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+\cdots+ab^{n-2}+b^{n-1}).$$

In particular when $n=2$ or $n=3$, we have

$$a^2-b^2=(a-b)(a+b)$$

and

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

respectively.

Proof. Both sides of the identity are 0 if $a=b$, and both sides of the identity are $-b^n$ if $a=0$. If $a\neq b$ and $a\neq 0$, then by the geometric sum formula we have

$$a^{n-1}+a^{n-2}b+\cdots+ab^{n-2}+b^{n-1}=a^{n-1}\sum_{k=0}^{n-1}\left(\frac{b}{a}\right)^k=a^{n-1}\cdot\frac{\left(\frac{b}{a}\right)^n-1}{\frac{b}{a}-1}=\frac{a^n-b^n}{a-b},$$

so the identity follows after multiplying $a-b$ on both sides. ■

When we encounter expressions that involve a **difference of square roots or n^{th} roots**, we may apply the above identity to get rid of such a difference. This technique called "**rationalization**" will sometimes be useful when we study limits and differentiation in chapters 2 and 3.

Example 0.33 (Rationalization) Let x and y be positive real numbers. Then we have

$$\sqrt{x}-\sqrt{y}=(\sqrt{x}-\sqrt{y})\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{x-y}{\sqrt{x}+\sqrt{y}}$$

and

$$\sqrt[3]{x}-\sqrt[3]{y}=(\sqrt[3]{x}-\sqrt[3]{y})\frac{(\sqrt[3]{x})^2+(\sqrt[3]{x})(\sqrt[3]{y})+(\sqrt[3]{y})^2}{(\sqrt[3]{x})^2+(\sqrt[3]{x})(\sqrt[3]{y})+(\sqrt[3]{y})^2}=\frac{x-y}{(\sqrt[3]{x})^2+(\sqrt[3]{x})(\sqrt[3]{y})+(\sqrt[3]{y})^2}.$$

Summary of Chapter 0

The following are what you need to know in this chapter in order to get a pass (a distinction) in this course:

- ✓ To recognize a **set** and its **elements**; **equality** of two sets
- ✓ Universal notations of some important sets: $\emptyset$, $\mathbb{R}$
- ✓ Intervals; **open intervals** (a, b), **closed intervals** $[a, b]$ and half-open intervals $(a, b]$, $[a, b)$
  - ⊙ To convert the set of numbers satisfying an inequality to the **interval notation**
- ✓ **Operations** on sets: intersection, union, complement

- ✓ Solving (real algebraic) **inequalities in one variable**, table of signs
- ✓ The **absolute value** of a real number
- ✓ Solving equations or inequalities that involve absolute signs

- ✓ The **Sigma notation** for sums
- ✓ Summation formulas, **geometric sum**
- ✓ Method of **rationalization**